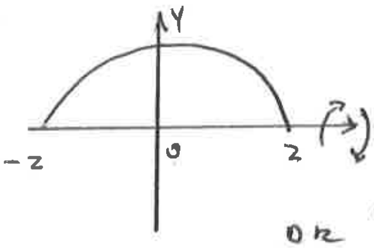
MATH 2460 EXAM 1

NAME Key - GRADE OUT OF 15 PTS

Answer the following questions correctly for a full credit. **NO DECIMAL.**

1. **(2pts)** Find the volume of the solid formed by revolving the graph of $y = \sqrt{4 - x^2}$ about the x-axis. (**Show** your work clearly)



$$V = \pi \int_{-2}^{2} y^2 \, dx = \pi \int_{-2}^{2} \left[\sqrt{4-x^2}\right]^2 dx$$

or $$V = 2\pi \int_{0}^{2} \left[\sqrt{4-x^2}\right]^2 dx = 2\pi \int_{0}^{2} 4 - x^2 \, dx$$

$$\vdots$$

$$= 2\pi \left[4x - \frac{1}{3}x^3\right]_0^2$$

$$= 2\pi \left[8 - \frac{8}{3}\right] = \boxed{\frac{32\pi}{3} \text{ unit}^3}$$

2. **(1pt)** **Choose ONE** of the following questions:(Either (a) OR (b) and clearly **show** your work!)

   (a) Evaluate $\displaystyle \frac{d}{dx} \int_{x^2}^{3} \sin(t^2) \, dt$

   (b) Find $F'(x)$ when $\displaystyle F(x) = \int_{x}^{x^2+4} \tan^2(t) \, dt$

a $\displaystyle \frac{d}{dx} \int_{x^2}^{3} \sin(t^2) \, dx = -\sin\left[(x^2)^2\right] 2x = -2x \sin(x^4)$

b $F' = \tan^2(x^2+4)\, 2x - \tan^2(x)$

3. **(2pts)** <u>**Choose ONE**</u> of the following questions:(Either (*a*) OR (*b*) and clearly <u>**show**</u> your work)

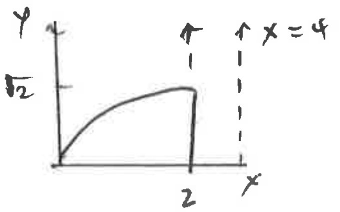
Consider the region R_1 which is bounded by the curve $y = \sqrt{x}$ and the lines $x = 2$ and $y = 0$ to answer the following either question (2pts).

(a) Use the disk/washer method to **set up** the integral that gives the volume of the solid formed by revolving the region R_1 about:

i. x–axis ii. y–axis iii. line $x = 2$ iv. line $x = 4$

(b) Use the shell method to **set up** the integral that gives the volume of the solid formed by revolving the region R_1 about:

i. x–axis ii. y–axis iii. line $x = 2$ iv. line $x = 4$



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(i) $\pi\int_0^2 [\sqrt{x}]^2 dx$ (ii) $\pi\int_0^{\sqrt{2}} [2^2-(y^2)^2] dy$

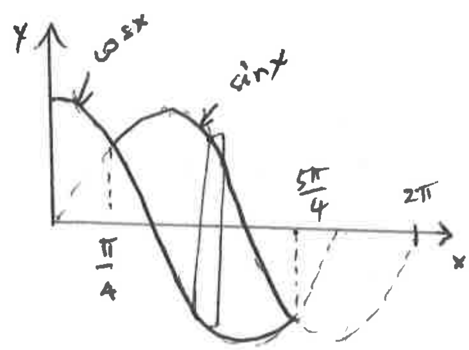
(iii) $\pi\int_0^{\sqrt{2}} (2-y^2)^2 dy$ (iv) $\pi\int_0^{\sqrt{2}} [(4-y^2)^2-2^2] dy$

b

(i) $2\pi\int_0^{\sqrt{2}} y(2-y^2)\,dy$ (ii) $2\pi\int_0^2 x\sqrt{x}\,dx$

(iii) $2\pi\int_0^2 (2-x)\sqrt{x}\,dx$ (iii) $2\pi\int_0^2 (4-x)\sqrt{x}\,dx$

4. **(1pt)** Find the area of the region bounded by $y = \sin x$ and $y = \cos x$ from $x = \pi/4$ to $x = 5\pi/4$.



$$A = \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)\,dx$$

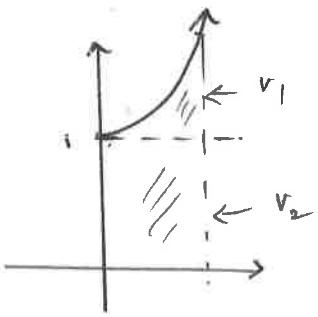
$$= \Big[-\cos x - \sin x\Big]_{\pi/4}^{5\pi/4}$$

$$= -\left[-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right]$$

$$\boxed{A = 2\sqrt{2} \ \text{unit}^2}$$

5. **(2pts)** Find the volume of the solid generated by rotating the region bounded by $y = x^2 + 1,\ y = 0,\ x = 0,\ x = 1$ about the y–axis using:

(a) the disk/washer (b) shell method. Which one is preferable?



$x = 1$

ⓐ Note: $V_2 = \pi (1)^2 (1) = \pi$ (right cylinder)

$$V_1 = \pi \int_1^2 \left[1^2 - \left(\sqrt{y-1}\right)^2 \right] dy$$

$$= \pi \int_1^2 (2-y)\,dy$$

$$= \pi \left[2y - \tfrac{1}{2} y^2 \right]_1^2 = \frac{\pi}{2}$$

So $V = V_1 + V_2 = \frac{\pi}{2} + \pi = \boxed{\frac{3\pi}{2} \text{ unit}^3}$

ⓑ $$V = 2\pi \int_0^1 x\left(x^2+1\right) dx = 2\pi \int_0^1 \left(x^3 + x\right) dx$$

$$= 2\pi \left[\tfrac{1}{4} x^4 + \tfrac{1}{2} x^2 \right]_0^1$$

$$= 2\pi \left[3/4 \right]$$

$$= \boxed{\frac{3\pi}{2} \text{ unit}^2}$$

Shell is clearly preferrable!

6. **(1pt)** Evaluate $\displaystyle\int_1^2 \frac{1}{x}\, dx$

$$= \left[\ln|x| \right]_1^2 = \ln(2) - \overset{0}{\ln(1)} = \boxed{\ln 2}$$

7. **(1pt)** Find the indefinite integral $\displaystyle\int 2x\cos(x^2)\, dx$

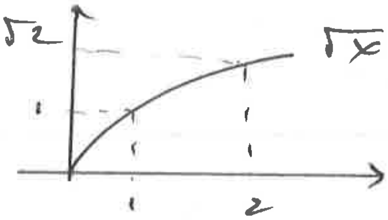
Let $u = x^2 \longrightarrow du = 2x\,dx \longrightarrow \int \cos(u)\, du$

$\longrightarrow [\sin u] + C = \boxed{\sin x^2 + C}$

8. **(3pts)** Use the Trapezoidal Rule and Simpson's Rule to approximate the value of the definite integral for $n = 2$. Round your answers to *four decimal places* and compare the results with the exact value of the definite integral. $\int_0^2 \sqrt{x}\, dx$ (**show** your work!)

(a) Trapezoidal Rule

$$1\left(\frac{0+1}{2} + \frac{1+\sqrt{2}}{2}\right) = \frac{2+\sqrt{2}}{2} \boxed{\cong 1.7071}$$



(b) Simpson's Rule

$$\frac{2(1)}{6}\left[0 + 4\cdot 1 + \sqrt{2}\right] = \frac{4+\sqrt{2}}{3} \boxed{\simeq 1.8047}$$

(c) Exact value

$$\int_0^{2} \sqrt{x}\, dx = \left[\tfrac{2}{3} x^{3/2}\right]_0^2 = \left[\tfrac{2}{3}\sqrt{x^3}\right]_0^2 = \frac{2}{3}\sqrt{8} = \frac{4\sqrt{2}}{3}$$

$$\boxed{\simeq 1.8856}$$

9. **(1pt)** Find the general solution of the differential equation and check the result by differentiation. (Use C for the constant of integration.) $\frac{dr}{d\theta} = 2\pi$.

$$r = \int 2\pi\, d\theta = \boxed{2\pi\theta + C}$$

10. **(1pt)** Solve the differential equation. $f'(x) = x^2 + 3,\ f(1) = 0$

$$f(x) = \int (x^2+3)\, dx = \frac{1}{3}x^3 + 3x^2 + C$$

Now $f(1) = \frac{1}{3} + 3 + C = 0 \longrightarrow C = -\frac{1}{3} - 3 = -\frac{10}{3}$

Hence $\boxed{f(x) = \frac{1}{3}x^3 + 3x - \frac{10}{3}}$